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Peak-to-Average Power Ratio Reduction Techniques in OFDM: A Review and Challenges

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***Abstract*—High value of Bits/second and Bits/second/hertz are very much essential to support the requirements of current and future mobile users. With Orthogonal Frequency Division Multiplexing (OFDM), these parameters are achieved very well as per the requirement, but at the cost of increased value of Peak- to- Average Power Ratio (PAPR). High PAPR increases the power consumption, thereby reduces the battery life. This article presents review and analysis of the techniques used for PAPR reduction in OFDM. Also, we have suggested the scope and challenges in PAPR reduction.**

***Keywords*— OFDM, PAPR, LTE.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a basically multi carrier communication [9]-[10] system. Due to advancements in baseband processor architecture [8] with in built math co-processor, OFDM implementation becomes practical. Long Term Evolution (LTE) [9] uses OFDM at core level. In OFDM, placement of carriers are orthogonal to each other. To support high value of Bits/second, data needs to be transmitted using more number of such orthogonal sub-carriers. When number of carriers are more, it increases the ratio of peak power to average power [1]. This is the conflict present in multi carrier communication system design. Hence, high value of Peak-to-Average Power Ratio (PAPR) is a very challenging problem [10]. PAPR governs signal amplitude variations. In wireless communication system, power amplifiers are present before transmitting antenna. They are operated near to saturation region [2] to have power efficient system. But high PAPR demands large operating linear region to accommodate the entire OFDM time domain signal. If power amplifiers are operated in large linear region, system becomes power hungry. Hence operating region of power amplifier need to keep very close to saturation region. When signal with high PAPR facilitated through such a small operating region, clipping of signal occurs. This leads to signal distortion thereby introducing more errors [3]. Hence reduction of PAPR without compromising the other system parameters such as Bit Error Rate (BER), Computational Complexity [2], [4], [5] and Memory requirements [3] becomes very much essential in multi carrier communication environment.

II. PAPR PRELIMINARIES

Definition of PAPR is given below [6]:

$$\mathrm{PAPR} = \frac{\mathrm{P_{peak}}}{\mathrm{P_{average}}} \tag{1}$$

Increase in average power present in the denominator of Equation (1), should never be done for reduction of PAPR. PAPR can be measured either in continuous time or in discrete time [6].

$$\mathrm{PAPR}=\frac{[max|x_n|^2]}{E[|x_n|^2]} \tag{2}$$

where xn = Complex sequence at the output of IDFT block, and E [·] = Averaging operator. For measurement of PAPR, Complementary Cumulative Distribution Function (CCDF) is used. When discrete sequence is considered for PAPR measurement then oversampling factor must be at least 4 as per the literature reported earlier [6]. Then only PAPR measurement is accurate. Basically, CCDF indicates the probability that PAPR will cross the set threshold value. CCDF graphs are used as a performance indicator for PAPR reduction technique.

III. CLASSIFICATION OF PAPR REDUCTION TECHNIQUES

PAPR reduction techniques are broadly classified into two major categories-

1) Signal Distortion Techniques
2) Signal Scrambling Techniques

Signal distortion techniques are- Peak Windowing, Envelope Scaling, Peak Reduction and Clipping followed by filtering. Whereas widely used and favorable signal scrambling techniques are- Block coding with error correction, Selected Mapping (SLM), Partial Transmit Sequences (PTS) and Interleaving. Now we will discuss the techniques from both the group in next section.

IV. Signal distortion techniques

PAPR reduction techniques from this category are generally not preferred. Because they causes degradation of other system parameters [16], [23] such as spectrum distortion, increase in BER, out of band radiation. But with these techniques PAPR can be drastically reduced.

A. *Peak Windowing*

In this, OFDM time domain signal is passed through filter. The peak from OFDM symbol is eliminated by multiplying large signal peaks with non-rectangular window [23]. Window must be as narrow as possible otherwise it affects many signal samples, which increases the BER value at transmitter itself.

B. *Envelope Scaling*

Based on total amplitude variations of OFDM symbol, proper scaling factor is decided and entire OFDM symbol is scaled down to reduce the overall PAPR value. This technique is better than peak windowing. But selection of scaling factor is a very crucial part of this technique. Scaling factor depends on operating range of High Power Amplifier (HPA). The scaled signal should fit inside the operating range of HPA.

C. *Peak Reduction*

In this, amplitude of maximum peak of time domain signal is reduced resulting in signal distortion. Note, in peak windowing, we completely removes the entire peak. So rise in BER is less as compared to that of peak windowing technique. This also distorts the signal spectrum.

D. *Clipping and Filtering*

In this technique, threshold for clipping [16], [19] is set based on HPA operating range. Clipped signal is then filtered to control the interference and to shape the signal spectrum.

Overall, these techniques reduces the PAPR at the cost of distorted OFDM spectrum at transmitter side itself. Receiver should have a strong error correction capability. This increases the receiver computational complexity and cost.

V. Signal Scrambling techniques

In this techniques, signal distortion is avoided. Basic principle used in this category is to scramble the data with different poly phase sequences [30] and make the data more random. More randomness causes more reduction in PAPR [1], [3]. In following sub-sections scrambling techniques for PAPR reduction are discussed.

A. *Block Coding with Error Correction*

Fig. 1 shows basic OFDM transmitter [9]. To have the error correction, various channel coding algorithms are used. This is a Forward Error Correction (FEC). FEC adds the redundant bits. Then this bit stream is passed to the mapper which is serial to parallel convertor (S/P). Output of S/P is fed to IDFT block which generates the OFDM symbol in time domain at its output. Length of IFFT decides the number of sub-carriers. PAPR reduction techniques are applied before HPA block. After IFFT, OFDM signal is pulse shaped, then modulated with RF carrier F_c. Then its power level is increased as per the design using HPA and finally transmitted using antenna.

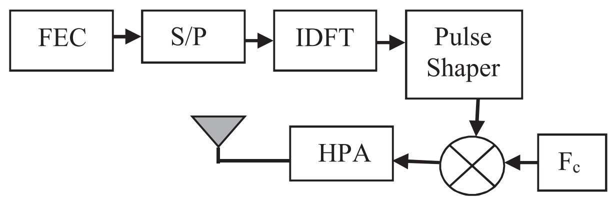

Fig. 1. Basic OFDM Transmitter

Table I provides the value of PAPR at the output of Binary Phase Shift Keying (BPSK) mapper in OFDM [1]. From table I, it is clear that, 4 data sequences [1,1,1,1], [1,-1,1,-1], [-1,1,-1,1] and [-1,-1,-1,-1] has PAPR value of 6 dB. These data sequences has large PAPR value compared to other data sets. This is because they have well defined pattern in it. We can reduce the PAPR by blocking the transmission of such sequences and allowing the codewords with less PAPR. But this introduces the error because of blocking of codewords with high PAPR. Hence sequences with well-defined pattern increases PAPR. In other words, we can say that sequences with more randomness has less PAPR. Thus, coding schemes also decides the PAPR. We should select the coding techniques such that resulting codewords should possess good randomness and also error correction capability should be maintained [2]. But, this is very difficult and challenging problem to search for such good codes. With increase in sub-carriers, computational complexity for searching good codes [14]-[15] also increases drastically.

Table I. Discrete Sequences and PAPR values

Sequence	PAPR (dB)	Sequence	PAPR (dB)
[1,1,1,1]	6.0	[-1,1,1,1]	2.3
[1,1,1,-1]	2.3	[-1,1,1,-1]	3.7
[1,1,-1,1]	2.3	[-1,1,-1,1]	6.0
[1,1,-1,-1]	3.7	[-1,1,-1,-1]	2.3
[1,-1,1,1]	2.3	[-1,-1,1,1]	3.7
[1,-1,1,-1]	6.0	[-1,-1,1,-1]	2.3
[1,-1,-1,1]	3.7	[-1,-1,-1,1]	2.3
[1,-1,-1,-1]	2.3	[-1,-1,-1,-1]	6.0

B. Partial Transmit Sequence (PTS) Technique

Fig. 4 shows basic schematic of PTS technique [4]. In this, input data of length N is divided into subblocks.

1	-1	1	-1	1	-1	1	-1

↓

1	-1	0	0	0	0	0	0

+

0	0	1	-1	0	0	0	0

+

0	0	0	0	1	-1	0	0

+

0	0	0	0	0	0	1	-1

Fig. 2. Creating sub-blocks in PTS technique

Consider the original data block given below. It has

$$X = [1, -1, 1, -1, 1, -1, 1, -1]$$

PAPR value of 9 dB without PTS. This PAPR is calculated using discrete formula of PAPR (2) described in section II of this article. Fig. 2 shows partitioning of original data X into 4 sub-blocks. Original data sequence has pattern in it, so it results large PAPR value. Now, these 4 sub-blocks are processed through the IDFT blocks. The sub-carriers in each sub-blocks has given proper phase weights. The phase factors selection is such that it will reduce the overall PAPR when all sub-blocks are combined. The length of IDFT should be minimum NL. L=4 is to be used as an oversampling factor [6]. N is number of sub-carriers to be used. This is to have proper measurement of PAPR. The phase factors are selected from the set of {+1, -1}. From table I, it is quite obvious that, phase factors of {+1, -1, -1, -1} results low PAPR. For phase factors, poly phase sequences [30], which are complex roots of unity can also be used.

Modified data block obtained after multiplying with these phase factors is shown in Fig. 3 below:

1	-1	0	0	0	0	0	0

+

0	0	-1	1	0	0	0	0

+

0	0	0	0	-1	1	0	0

+

0	0	0	0	0	0	-1	1

↓

1	-1	-1	1	-1	1	-1	1

Fig. 3. Modified data block after combination of phase weighted sub-blocks

First sub-block is multiplied by phase factor of +1, second sub-block is multiplied by phase factor of -1, third sub-block is multiplied by phase factor of -1 and fourth sub-block is multiplied by phase factor of -1. Now, all these phase weighted sub-blocks are combined to result modified data block. Now, the PAPR of modified data block is 5.2 dB. Thus, with this partial sequences, PAPR of original data block is reduced from 9 dB to 5.2 dB. This is a drastic reduction in PAPR. In this technique, PAPR reduction depends upon the number of sub-blocks and phase factors used for weighting the sub-carriers in each sub-blocks. We need to perform optimization while selecting the phase factors. We are selecting the phase factors which results reduction in PAPR after combining all partial sequences coming from sub-blocks. Also number of IDFT operations increases with increase in number of sub-blocks. Exhaustive search operation is required to figure out the set of good phase factors that results in PAPR reduction. Thus, using PTS scheme PAPR reduction is achieved at the cost of increased computational complexity [12].

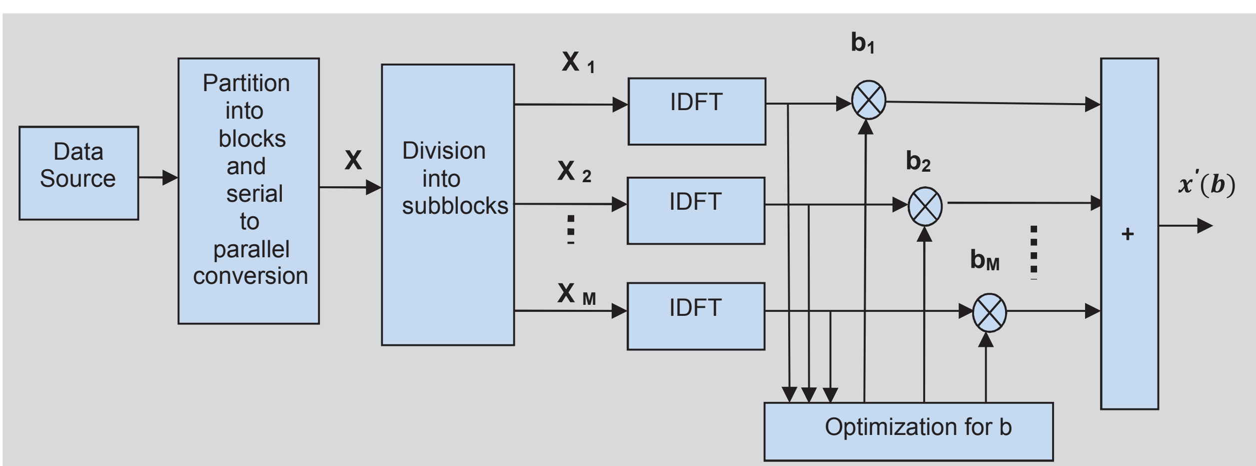

Fig. 4. Basic Schematic of PTS technique

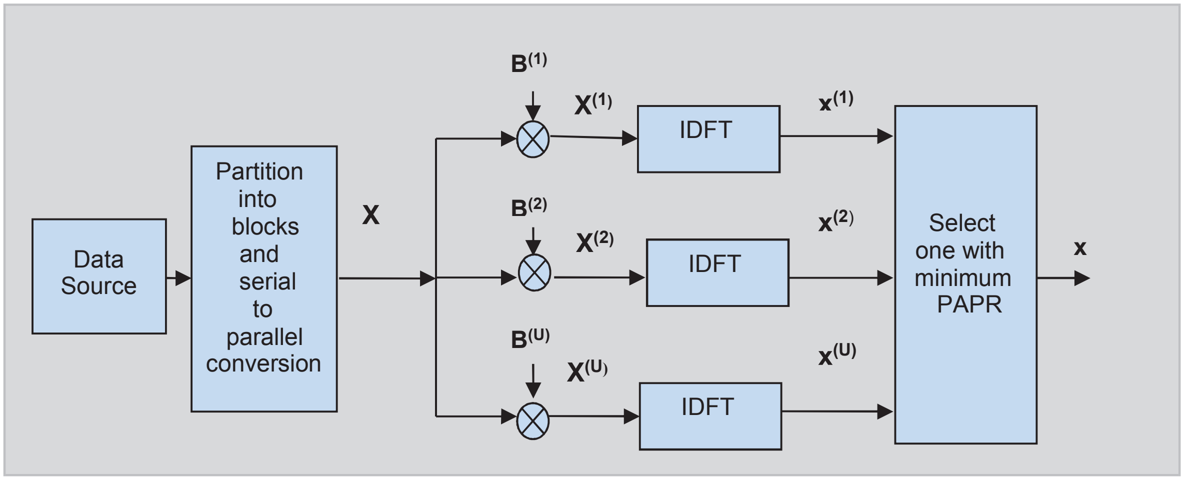

Fig. 5. A Block Diagram of the SLM technique

C. Selected Mapping (SLM) Technique

The block diagram of SLM technique [5]-[6] is shown in Fig. 5. In SLM, data is not converted into sub-blocks. Instead entire data block X, is multiplied by phase factors denoted by B. The number of phase factors are indicated by U. The length of each phase factor is same as that of data block X. In this technique, transmitter is generating U different data blocks by multiplying original data block X with U number of different phase factors and all generated U data blocks are representing the same information as that of X.

$$X^{(U)} = X.\,B^{(U)} \qquad (3)$$

Where, X = Original Data block

$B^{(U)}$ = U^{th} phase factor

$X^{(U)}$ = U^{th} different data block representing the information same as that of X

All such $X^{(U)}$ data blocks are passed through U number of IDFT block to generate U number of OFDM time domain signal denoted by $x^{(U)}$. Out of U number of such OFDM symbol representing the same information, symbol that has less value of PAPR is selected for final transmission.

Consider, Original data X = [1, -1, 1, -1, 1, -1, 1, -1].Take number of phase factors U=4. PAPR value of original data X is 9 dB without SLM. Let us consider the four phase factors are as $B^{(1)}$ = [1, 1, 1, 1, 1, 1, 1, 1], $B^{(2)}$ = [–1, –1, 1, 1, 1, 1, 1, –1], $B^{(3)}$ =[–1, 1, –1, 1, –1, 1, 1, 1], and $B^{(4)}$ = [1, 1, –1, 1, 1, –1, 1, 1]. Table II shows 4 different data blocks $X^{(1)}$, $X^{(2)}$, $X^{(3)}$, $X^{(4)}$ representing the same information as that of X generated using 4 different phase factors $B^{(1)}$, $B^{(2)}$, $B^{(3)}$, $B^{(4)}$. Now, PAPR values of this four different data blocks $X^{(1)}$, $X^{(2)}$, $X^{(3)}$, and $X^{(4)}$ are 9 dB, 4.65 dB, 3 dB and 5.16 dB respectively. Thus, out of these four data blocks $X^{(3)}$ has lowest PAPR of 3 dB. With SLM, PAPR of original data block X= [1, -1, 1, -1, 1, -1, 1, -1] is reduced from 9 dB to 3 dB. This reduction is very large as compared to PTS technique seen in earlier sub topic of this article.

TABLE II. DIFFERENT SETS OF ORIGINAL DATA FOR U=4

Phase Facto $B^{(U)}$	$X^{(U)} = X.\,B^{(U)}$
$B^{(1)}$ = [1, 1, 1, 1, 1, 1, 1, 1]	$X^{(1)}$ = [1, -1, 1, -1, 1, -1, 1, -1]
$B^{(2)}$ = [–1, –1, 1, 1, 1, 1, 1, –1]	$X^{(2)}$ = [-1, 1, 1, -1, 1, -1, 1, 1]
$B^{(3)}$ =[–1, 1, –1, 1, –1, 1, 1, 1]	$X^{(3)}$ = [-1, -1, -1, -1, -1, 1, 1, -1]
$B^{(4)}$ = [1, 1, –1, 1, 1, –1, 1, 1]	$X^{(4)}$ =[1 , -1, -1, -1, 1, 1, 1, -1]

So in this example, $X^{(3)}$ is selected for transmission. But, along with this data block we also need to transmit the information of phase factor that has resulted reduction in PAPR. For this example, $B^{(3)}$ has resulted lowest PAPR compared to other phase factors. As U=4 phase factors are used, $\lfloor \log_2 (U) \rfloor = \lfloor \log_2 (4) \rfloor$ = 2 bits are required to transmit the phase information. With these bits only, receiver is able to decide the phase factor that needs to be used for recovery of original data block at the receiver. This is the overhead present in the SLM. The transmission of overhead bits called as side information can be eliminated by using modified SLM technique [11]. Also searching the data block having less PAPR and number of IDFT operations involved, number of phase factors (U) in SLM increases the computational complexities. Some solutions have been reported in earlier literature to reduce the computational complexity of SLM [5].

D. Interleaving Technique

Just like a SLM, in interleaving technique also different OFDM data symbols representing same information as that of original data symbol are obtained. But, here phase factors are not used. Instead, N permutation functions are used for performing the interleaving operation. With these functions, reordering of original OFDM symbol is performed to generate N number of permuted OFDM symbols indicating same information [3], [31]. Out of N number of permuted blocks, the block that has less PAPR is selected for transmission. For this, N number of IDFT blocks are used. Receiver uses the same permutation function which has resulted OFDM symbol with less PAPR, for extracting the original OFDM symbol. But, for this, $\lfloor \log_2 (U) \rfloor$ number of extra bits needs to be transmitted. With these bits, receiver determines which permutation function is to be used from the set of N permutation functions. Number of interleaver and their design decides percentage of PAPR reduction.

VI. SCOPE AND CHALLENGES

Today, almost all communication systems like fourth generation (4G), LTE are making use of multi-carrier signalling approaches for achieving high data rate. Multi-carrier approach is also playing very important role in designing future high data rate communication [8]-[10] systems such as fifth generation (5G) mobile communication. But, this approach has one major problem of high PAPR. This leads to power inefficient systems. So reducing PAPR, thereby increasing battery life is still very challenging problem in wireless communication environment.

- The existing PAPR reduction techniques are having some other drawback associated with them which are summarized below:
  Large computational complexity [2]-[7] due to use of number of IDFT blocks present in PTS, SLM and Interleaving techniques, Number of multipliers used in PTS and SLM for phase sequences, Searching the codeword, Phase factors and permutation function from the set that results reduction in PAPR. Large computational complexity increases the machine cycle [8] requirements for implementing the algorithms for PAPR reduction in real time processor. Execution of PAPR reduction algorithms within well-defined time is equally very important.
  BER increases [16], [19], [23] in all signal distortion techniques for PAPR reduction. To solve this problem of increased BER, communication system should be incorporated with strong error correcting and decoding algorithms which leads to increase in computational complexity.
  Memory requirement increases in coding [1], [2], [7], [21], PTS [4], [12], SLM [3], [5], [6], [11], [24], and interleaving [3] techniques of PAPR reduction. Because in these techniques, large memory size is required for look up tables to store the sets of codewords, poly phase sequences [17], [29], different data blocks representing the same information, interleaved data. From these look up tables, entry that has less PAPR is selected for transmission.
- Designing of good sequences [14], [15], and [29] for PAPR reduction is a very challenging problem.

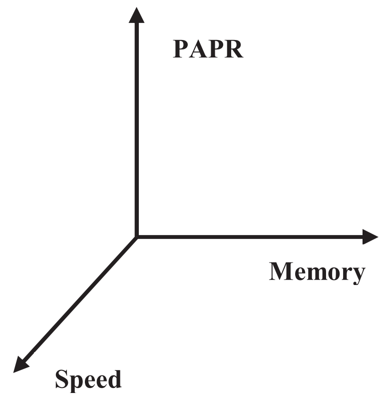

Fig. 6. Three dimensional Optimization for improving the performance of PAPR reduction techniques

Performance of PAPR reduction algorithm should be tested with respect to the amount of PAPR reduction, their speed of execution on real time processor and memory requirements [8]. From Fig. 6, it is clear that, investigating a technique that reduces the PAPR drastically with high speed of operation and minimum memory requirement is a challenging task. So while selecting the source coding, channel coding, modulation techniques one has to also look for variation in PAPR for the selection along with the error correction and data rate capability. The data sequences generated after applying the coding techniques, if possesses less randomness, then it results high value of PAPR. Randomness in discrete sequences can be governed by auto correlation [28], [30] levels.

VII. CONCLUSIONS

High PAPR is a very critical problem in Multi Carrier Communication System. PAPR is highly depends on the bit pattern. Therefore, designing of good PAPR reduction code is still a challenging open problem. Also there is wide scope for providing the optimum solution for the existing trade-offs. The term good code is itself very relative. Hence, defining the good PAPR reduction code with statistical measurement considering the rest communication system parameters is also very challenging problem. Designing PAPR reduction codes with error correction capability is highly appreciable.

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