Thermo-mechanical analysis of ventilated discs used in disc brakes by various design parameters: a review

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Abstract: Disc brake systems play a pivotal role in the automotive industry, ensuring both safety and performance in vehicles across the globe. This paper presents a thorough review of thermo-mechanical analyses conducted on disc brake systems, aiming to consolidate existing knowledge in this vital area of research. The review examines the basic operating principles of disc brakes, focusing on the mechanisms that produce and dissipate heat during braking as well as the analytical and computational approaches used to analyse the thermal behaviour of disc brakes. Furthermore, this review scrutinises the mechanical aspects of disc brakes, encompassing their structural mechanics, stress analysis, and material selection. The influence of design parameters, such as disc geometry and material, on mechanical performance is also discussed. Furthermore, it identifies potential areas for future research, including advancements in simulation techniques, materials, and design strategies, with the ultimate goal of enhancing disc brake safety, reliability, and performance.

Keywords: thermo-mechanical; brake disc; optimisation; temperature; stress; deformation.

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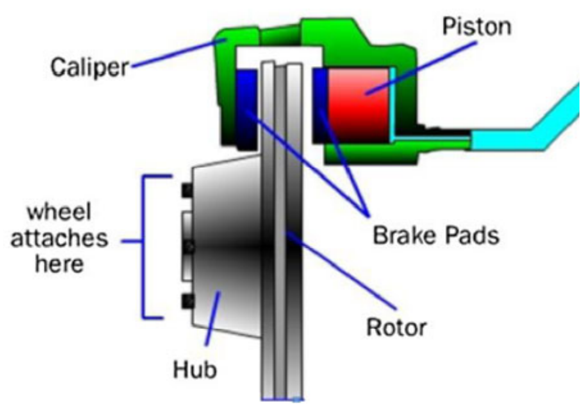
1 Introduction

A disc brake can be a floating or fixed caliper. The floating caliper type of disc brake assembly consists of components such as a caliper, brake pad rotating disc, piston to operate the pads, carrier, and guide pin, as illustrated in Figure 1. The piston uses pressurised oil to move the braking pads. The caliper houses the piston and pads. The most important requirement for effective braking is the application of uniform pressure on both pads so that they can exert uniform pressure on the disc. This results in uniform wear of brake pads and uniform brake temperature. Non-uniform pressure distribution results in uneven wear and reduces the life of the brake pad. It may also generate vibration-induced noise known as brake squeal.

Figure 1 Disc brake assembly (see online version for colours)



Source: Nathi et al (2012)

Ventilated disc brakes are preferred over solid disc brakes or drum brakes due to their high heat dissipation characteristics. Turbulence is created in ventilated discs, which results in heat dissipation at a higher rate. The advantage of a ventilated disc brake is that it decreases the thermal stresses in the disc due to the equal distribution of material. The geometry of the disc and the ventilation passage greatly influence the magnitude of the thermal stress.

Many types of brake discs are invented and used in vehicles to improve the cooling characteristics of the ventilated brake discs. Vanes, either radial or curved, are placed between two thick discs, which act as heat dissipation elements. Such discs suck cold air into the ventilated channels. In comparison to solid discs, cooling performance is enhanced due to the ram effect that is achieved due to the motion of the vehicle and the increase in effective heat transfer area. Ventilated brake discs with curved vanes have limited use since they are unidirectional, while radial vanes and pin-finned brake discs are widely used as they are bidirectional and can be mounted on any tyre of a vehicle.

2 Development history

A lot of research has been carried out on disc brakes which date back to the early 20th century but it was boosted in the late 70s after the invention of computers. Spurr (1961) has presented the theory of brake squeal. Rusnak et al. (1970) used an IBM 360/50 digital computer to perform the calculations. Limpert (1975) presented the equations for calculating heat-related parameters like heat flux and convective heat transfer coefficients. To date, these equations proved to be useful. After the invention of powerful computers and the finite element calculation program, the time required for the design and cost required for the development was reduced drastically (Timtner, 1979). Fukano and Matsui (1986) were some of the earliest few researchers who used computer simulation techniques to determine temperature characteristics in actual running conditions and thermal strength performance of the rotors. Various commercial software's were then invented and used for predicting the thermal, structural, and thermo-mechanical behaviour of the discs in the 90s. ANSYS and ABAQUS gained a lot of popularity. The results obtained from the numerical analysis were validated using experimentations. Several studies were reported in the literature regarding experimentations for calculating the temperature rise, pressure, and deformation (Kubota et al., 2000; Grivc et al., 2019; Towoju, 2019; Kumar et al., 2020; García-León et al., 2021). At the same time with the invention of new materials, the disc brake performance is improved by selecting materials with proper thermo-physical properties balancing their weight and cost. Optimisation techniques are also employed to improve thermal and structural performance of the disc rotor by considering different geometrical and operating parameters.

This review article is composed of different sections, which includes discussion on, calculation of heat flux and convective heat transfer coefficients, design parameters, discs based on their geometry (ventilation patterns) and materials, optimisation of disc design. Performance improvement using CFD, challenges and future direction and finally the conclusion is presented.

3 Calculation of heat flux and convective heat transfer coefficients

When brakes are applied in order to reduce the speed of the vehicle or to stop it, the kinetic energy possessed by the vehicle is converted into heat due to the friction between the disc and the pad. This gives rise to excessive temperature and large thermal stresses are induced. In thermo-mechanical analysis, the accuracy of the results depends on heat entering into the system which is given in the form of heat flux and heat dissipated governed by convective heat transfer coefficient. The heat flux entering the disc and convective heat transfer coefficient decides the maximum temperature reached. Considering the vehicle dynamics and that 60 % of the load is taken by front brakes, the force acting on each of the brake rotors F_{rotor} and instantaneous heat flux into the rotor $Q_{in}(t)$ are calculated as

$$Frotor = \frac{(30\%)\frac{1}{2}Mv_0{}^2}{2.\frac{r_{rotor}}{r_{tite}}\cdot\left(v_0 \cdot t_{stop} - \frac{1}{2}\left\{\frac{v_0}{t_{stop}}\right\}t_{stop}{}^2\right)}$$

Instantaneous heat flux into the rotor is calculated using the relation

$$Q_{in}(t) = \left(F_{rotor}\right)v_{rotor}(t) = F_{rotor}\frac{r_{rotor}}{r_{tyre}}\left(v_0 - \left\{\frac{v_0}{t_{stop}}\right\}t\right)$$

where,

M mass of the vehicle (kg)

v_0 initial velocity (m/s)

t_{stop} time to stop (s)

r_{rotor} effective radius of the rotor (m)

r_{tire} radius of the wheel (m).

The above formula is then integrated within the time interval and is divided by the brake pad area in order to calculate the heat flux (Mackin et al., 2002; Park et al., 2022). Another method for calculating the heat flux entering the disc is given by Belhocine et al. (2021, 2016).

$$q0 = \frac{1-\varnothing}{2}\frac{mgv_0z}{2A_d\varepsilon_p}$$

where

z a/g is the braking efficiency

g gravitational acceleration (m/s^2)

a vehicle retardation (m/s^2)

A_d disc surface area swept by brake pad (m^2)

m mass of vehicle (kg)

v_0 initial speed of the vehicle (m/s)

ε_p factor load distribution on the disc surface

$\varnothing$ rate distribution of the braking forces between the front and rear axle.

Another model of heat flux is presented by Dubale et al. (2021), and Tauviqirrahman et al. (2023).

$$q_{2(r,t)} = \frac{\Phi_0}{2\pi}\mu\varpi p_{max}r_3\omega_{0*}\left(1-\frac{t}{tb}\right)$$

where

q_2 heat flux on disc, W/m^2

μ friction coefficient

ϖ heat partition coefficient

ϕ_0 arc angle pad, deg

ω_0 wheel initial speed, rad/s

p_{max} maximum pressure, N/m^2

r_3 inner radius in pad, mm.

Talati and Jalalifar (2008) used two models for calculating the heat generated due to friction, namely macroscopic and microscopic. The first model is based on the law of conservation of energy, while in the second one, different parameters such as the velocity of the vehicle, brake dimensions and geometry of the brake system, braking duration, and disc and pad materials are considered. Uniform pressure and uniform wear conditions are considered for calculating the heat flux in the model. These two models accurately predict the amount of heat produced. The heat flux calculated is a function of time and space variables.

The heat transfer coefficient has a significant effect on the results of the numerical simulation. To ensure that the simulation analysis errors are well within their limits, it becomes important to determine their value precisely. ANSYS CFX can be used to determine heat transfer coefficients for a given disc at different vehicle speeds and temperatures (Pevec et al. 2012). The heat transfer coefficient is a parameter relates with the velocity of air and the shape of brake disc, and many other factors. In different velocity of air, the heat transfer coefficient in different parts of brake disc changes with time. The average value of the coefficient h 'Wall heat Transfer Coefficient', variable with time using ANSYS CFX is calculated (Belhocine et al., 2016, 2021).

The cooling rate of solid and ventilated part of the disc is different. For the solid part of the ventilated disc, the convection heat transfer coefficient associated with laminar flow can be approximated by

$$h = 0.70(ka / D)\,\mathrm{Re}^{0.55}$$

where,

D is the outer diameter of disc

Re is the Reynolds number

k_a is the thermal conductivity of air.

For ventilated portion, i.e., for straight vane of disc in the laminar flow condition Re < 104, the heat transfer coefficient inside the vanes of the brake disc is

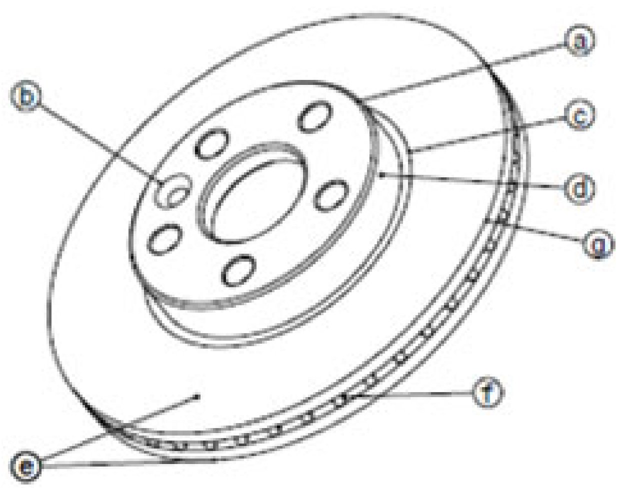
$$h_R = 1.861(\mathrm{Re}.\mathrm{Pr})^{\frac{1}{3}} \left(\frac{dh}{l}\right)^{0.33} \left(\frac{ka}{dh}\right)$$

where l is length of cooling vane, Pr is the Prandtl number, and Re = $(\rho_a d_h/\mu_a)V_{average}$. The hydraulic diameter d_h is defined as the ratio of four times the cross-sectional flow area (wetted area), divided by the wetted perimeter (Hwang and Wu, 2010).

4 Design parameters affecting performance of disc brake

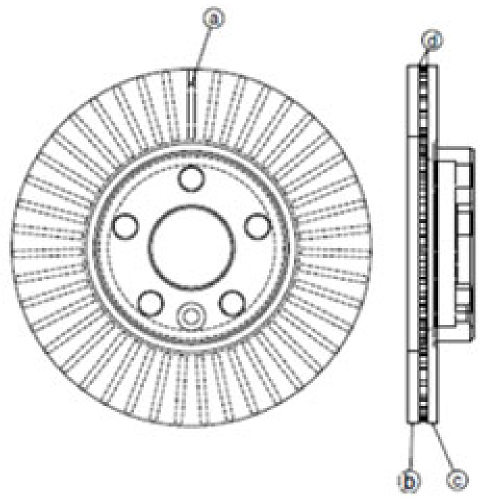
Figure 2 and Figure 3 shows various design parameters that can affect the performance of the ventilated disc. A ventilated disc consists of ventilation blades sandwiched between inboard and outboard discs. These ventilation blades help to dissipate heat. The disc is cooled with air circulation by the ventilation blades due to the action of centrifugal force. Number of ventilation blades and thickness of the blades also have impact on the cooling of the disc. The ventilation blades can be straight (radial) or curved.

Figure 2 Ventilated brake disc showing important design parameters (a) inner diameter of the disc (b) fixing the hub (c) cooling channel (d) bell (e) friction tracks (f) ventilation blades (g) outer diameter of the disc



Ventilation gap is another important design parameter which is focused by the researchers for the optimum performance. This gap is responsible for sucking the air and circulating it in between the cooling channels. Friction tracks are the tracks where the pad is pressed against the rotating disc. They are subjected to large force and maximum heat is generated here due to friction. Holes, slots/slits, and combination of these two are provided on the friction tracks for increasing the surface area for convection. The holes and slits have advantages of improving the braking performance by improving the frictional force, heat dissipation, dust and gas emission, disc deformation and fading, and weight reduction. However stress gets concentrated around the holes and becomes one of the points for thermal crack generation. Bell is the portion of the disc through which the disc is attached to the hub. Cooling channels are provided for effective cooling. External and internal diameters of the disc are decided by space constraints.

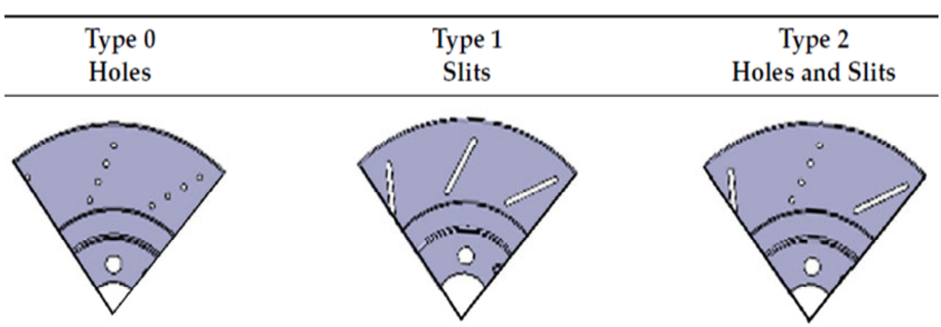
Figure 3 Two dimensional representation of ventilated disc (a) ventilation blade thickness (b) outboard disc thickness (c) inboard disc thickness (d) ventilation gap



5 Discs based on geometry (ventilation patterns) and materials

The technique of drilling holes and cutting slits on the friction surface of the disc to increase the area exposed to air and enhance the heat dissipation capacity was adapted by Park et al. (2022). A finite element model was prepared to evaluate the performance of the disc. Structural and thermal analysis was carried out using the developed model.

Figure 4 Different patterns on the disc surface



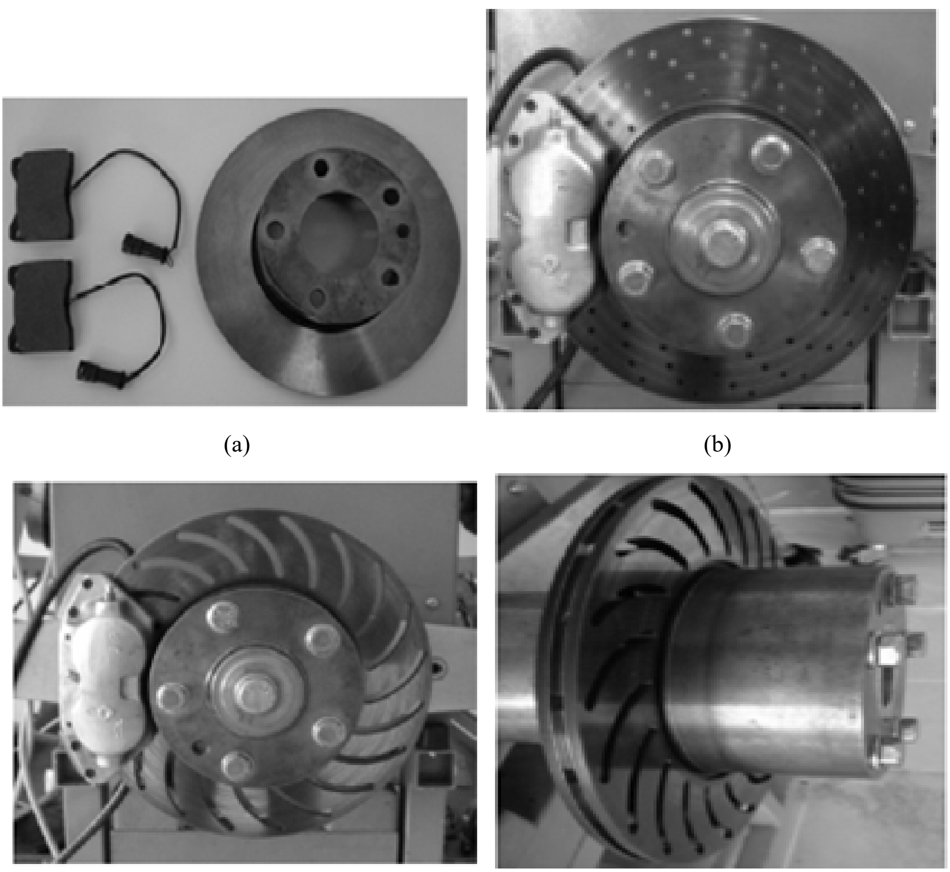
Source: Park et al. (2022)

As shown in Figure 4, type 0 is the disc with holes, type 1 is the disc with slits cut, and type 2 is the disc with drilled holes and cut slits. Along with improved heat performance, it was found that these patterns were prone to stress concentration. To overcome this issue, optimisation was carried out.

Duzgun (2012) considered three ventilated discs with different patterns and designs and investigated their thermal behaviour using FEA and experimental analysis. The different designs considered are shown in Figure 5. The analysis results of the three new designs were then compared with the results of the solid disc. It was concluded that, due to the provision of ventilation patterns, heat generation was reduced by 24% compared to a solid disc. The FEA results obtained were validated by experiments and were found to be in the acceptable range. Also, in comparison to the other two ventilated patterns, the disc with cross slots and side groves was to be the better one.

Afzal A (2020) described comparable work on discs with various geometrical patterns, as depicted in Figure 6. Thermal and structural analysis was performed using ANSYS. The temperature rise, total deformation, stresses induced, and contact pressure were determined for all three models developed. The results show that curved vanes performed better than straight vanes, while the thermo-mechanical behaviour of discs can be improved by providing holes and slots on the surfaces.

Figure 5 Different designs of discs (a) SL disc (b) CD disc (c) CS disc (d) CS-SG disc (see online version for colours)



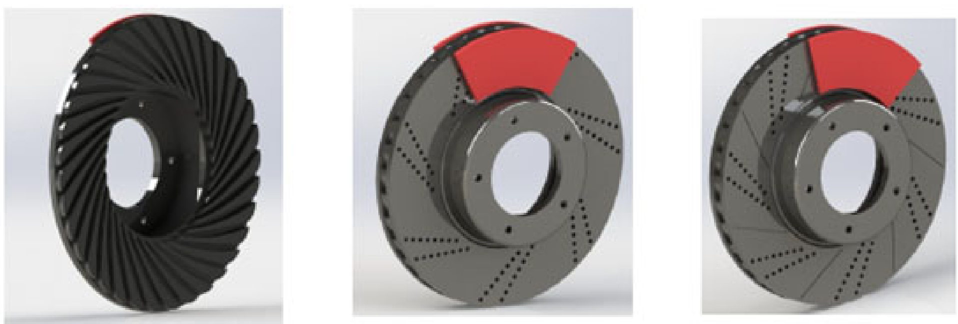
(a) (b) (c) (d)

Source: Duzgun (2012)

Dubale et al. (2021) carried out analytical and numerical analysis on three different types of disc profiles to study transient thermo-mechanical properties, as shown in Figure 7.

Analytical and numerical analysis is performed on the disc profiles, viz., solid, drilled, and grooved, for assessing different thermal parameters like the maximum temperature reached, thermal stresses, and deformation. As per the results obtained, it is clear that the maximum equivalent stress induced and maximum deformation in the grooved profile is less than those of the other two types. This is due to the fact that a grooved profile dissipates heat at a faster rate. Also, in a drilled-type profile, increasing the number of holes increases the heat transfer efficiency.

Figure 6 Discs with curved vanes, curved vanes with holes, and curved vanes with holes and slots (see online version for colours)



Source: Afzal (2020)

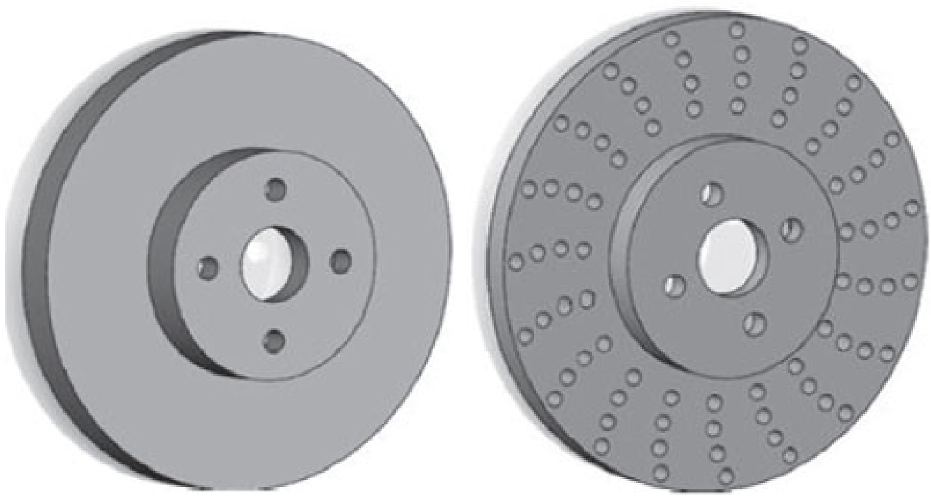
Dhir (2018) considered three designs of rotors. Disc rotor 1 is solid, while disc rotors 2 and 3 have an airfoil design with holes and slots, as shown in Figure 8.

Thermo-mechanical analysis was carried out, and the maximum temperature and life span were determined. In disc rotor 1, which has the maximum weight, the temperature rise is also higher than in the other discs. The temperature rise in the discs other than solids is less due to the larger surface area exposed to the surroundings. Other than temperature rise, the geometric variations in disc profiles also affect the total life span of disc brakes, but the effect is opposite. The solid disc rotor has more life and can withstand greater values of cyclic loads compared to other profiles.

Pranta et al. (2022) determined the stress and temperature distribution on different types of ventilated disc brakes with curved vanes, slots, and holes. The discs were modified in terms of shape and pattern. 3D models were prepared and analysed using FEA to determine the thermal and structural characteristics. Additionally, different materials were also analysed, like steel, cast iron, and CFRP laminates, and the best suitable material was found on the basis of the parameters analysed, like stress, temperature, and factor of safety.

García-León and Flórez-Solano (2017) analysed the dynamic behaviour of the disc brake system with the help of solid works simulation software. Three different brake discs from different vehicles were analysed. The first disc D1 of the vehicle, having a mass of 1,250 kg and a disc mass of 3.8 kg, the second disc D2 of the vehicle, having a mass of 1,950 kg and a disc mass of 4.25 kg, and the third disc D3 of the vehicle, having a mass of 2,250 kg and a disc mass of 6.1 kg, were considered. The geometry and ventilation channels for each disc were also different. From the results, it is clear that disc D1 is better in comparison to discs D2 and D3 in terms of working conditions such as speed and displacement in braking. It was concluded that the systems can be optimised by optimising the brake disc geometry and the ventilation channels.

Figure 7 Disc profiles (a) solid (b) drilled (c) grooved



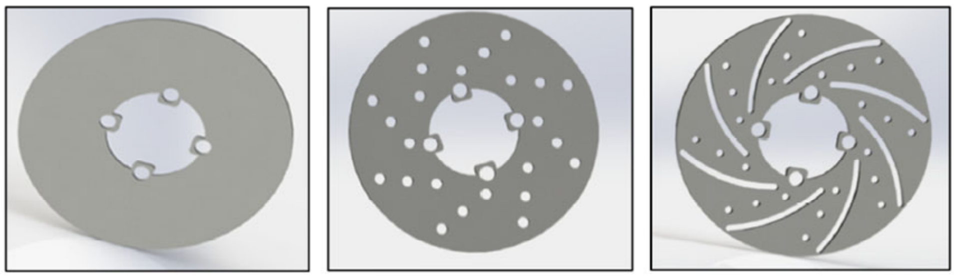
(a) (b)

(c)

Source: Dubale et al. (2021)

García-León et al. (2021) used ANSYS software to study five common automotive discs. A comparison between theoretical, simulation, and experimental data collected from earlier research is done to improve the geometry and enhance heat transfer. The results obtained from numerical analysis are then used to derive two alternative solutions for brake discs using N-38 ventilation blades used in aeronautic engineering. These blades showed improved heat dissipation, i.e., 23.8% more in comparison to the five geometries analysed. Finally, it was concluded based on the mathematical calculations and experiments that the amount of heat generated and its proper dissipation mainly depend on the geometrical parameters of the disc, the materials of the disc and pad, the weight of the disc and vehicle, and the conditions of operating the vehicle.

Figure 8 Solid disc, disc with holes, and disc with holes and slots



Source: Dhir (2018)

Kumar et al. (2020) determined the mechanical and tribological properties of different fibre-reinforced composites with the objective of finding a suitable material for rotating applications. Numerical analysis was done, and the results were validated experimentally using pin-on-disc apparatus under similar conditions. The epoxy-carbon-woven-prepreg-SiC composite material was found to be the one that can perform better when subjected to rotodynamic stresses after total deformation, shear stress, and equivalent stress were calculated.

Thermal overload causes brake fade and limits the maximum temperature in the brake components. Braking patterns can play an important role in avoiding brake fade problems. Towoju (2019) numerically investigated the braking patterns and their effect on temperature rise, which eventually leads to brake fade. A braking pattern that involves a total time of 5 seconds with two brakings and a time interval in between was considered. Different braking patterns were studied, and the pattern with the lowest peak temperature will avoid brake fade. The pattern, which involves two brakings of 1s and 3s with a time interval of 1s, proved to be the better one with a maximum temperature rise of 600 K.

Singh Chouhan and Tiwari (2020) studied different combinations of disc and pad geometry. Solid disc and ventilated disc with solid and slotted pad and ventilated disc with solid and slotted pad combinations were used for analysing the thermo-mechanical behaviour. Thermal and structural analysis for different combinations of disc and pad geometry was carried out using commercially available software. It is found from the obtained result that solid discs deform more, but maximum stresses are less. Ventilated discs show good thermal performance compared to solid discs.

Hariram et al. (2021) performed thermal and structural analysis on a ventilated brake disc and pad model with the objective of minimising the heat produced during the braking process. PTC Creo is used for modelling and the ANSYS workbench for analysis. Analysis was carried out on newly developed brake disc and pad profiles for determining parameters such as deformation, stress, maximum temperature reached, and heat flux. A braking cycle of 4.5 s was used for assessing the above parameters. Based on the results, the best possible design is suggested.

Many researchers have carried out research on materials that can be used for manufacturing the disc of the brake system. Heat dissipation in the surroundings is one of the important factors considered in the selection of the proper material. Seelam et al. (2021) considered gray cast iron and stainless steel as the materials of the brake disc of a high-speed car. The comparison of the two materials is done numerically by carrying out structural analysis and thermal analysis. From the analysis conducted on the discs for determining the thermal performance and life of the disc, it was clear that the material that dissipates heat at a faster rate performs better. Gray cast iron outperforms stainless steel as its life is longer due to its anti-fade property.

Chheda and Hattale (2020) analysed the material performance requirements for the disc materials to replace conventional cast iron material with other lightweight materials. For selecting the material, mechanical and thermal properties play an important role. Finite element analysis was carried out on identical discs with different materials under identical conditions. From the results, titanium proved to be the material that could perform efficiently.

Belhocine and Afzal (2020) performed finite element analysis on a solid and ventilated disc, considering different materials. Three different grades of cast iron were the materials. The purpose of the study was to compare solid and ventilated discs and also find a suitable material for the discs. Computational fluid dynamics was used for the determination of the heat transfer coefficient, which was later used to determine the maximum temperature in the further analysis. The results show that a ventilated disc dissipates heat at a faster rate than a solid disc. From a material point of view, cast iron FG15 is the one that has the maximum temperature.

Ahmad et al. (2021) studied the thermo-mechanical behaviour of four different materials used for manufacturing the disc. A 3D model of the brake disc was designed, and analysis was performed to determine the maximum temperature, heat flux, total deformation, and stresses. The conclusions made from the results obtained were as follows: aluminium-made discs show poor thermal performance as compared to the other three materials. Carbon-carbon and titanium alloys have the best performance from a heat dissipation and total deformation point of view. Balu and Rajendra (2023) investigated various materials for increasing braking efficiency and improving vehicle stability. Three types of materials and two types of profiles were analysed on different parameters, which include temperature, stress, and deformation. Aluminium composite material was found to be the optimal one for discs. Deformation in aluminium was less than in the other two materials for different mode shapes.

Table 1 Results for the two discs

Material	*Heat flux (W/mm^2)*	*Maximum deformation (mm)*	*Maximum stress (MPa)*
Aluminium alloy	8	0.19	180
Aluminium matrix nanocomposite	16.28	0.06	184

To increase the heat dissipation property and reduce the weight of the disc, Sivaprakasam et al. (2022) studied the thermo-mechanical behaviour of brake discs made of two different materials. The first disc is made from aluminium alloy, while the second was made from nanocomposite particles of aluminium to reduce the weight. The analysis was conducted using FEA under similar conditions of braking. From the results, it was found that the disc made from nanocomposite aluminium matrix performed better than the other. Heat flux was higher, which indicates faster cooling, and deformation was much less than in the other material. The quantitative data is shown in Table 1.

From the analysis results, the aluminium matrix nanocomposite material can be a replacement for the conventional materials used for the disc of a brake system.

Choudhary et al. (2023) conducted the thermo-mechanical analysis for determining the optimum material from three different materials. The materials considered for the study of disc properties were AISI4140, SS420, and AISI1020. In order to ensure that factors other than the qualities of the materials would not affect the results, both the thermal transient analysis and the structural static analysis were carried out under identical conditions. Analysis software ANSYS is used for the analysis, and SS420 is found to be the best material in terms of thermo-mechanical properties.

Al Riyami et al. (2023) designed the ventilated brake disc for heavy-duty cars with the objective of selecting a material that would perform better and reduce damage due to high temperatures. Two different disc materials, GCI and cast carbon steel, are considered for the analysis. To increase the surface area exposed to air for cooling, holes are drilled on the surface of the disc. The numbers of vanes are also altered for both materials. Temperature and structural behaviour are analysed using modelling and analysis software. It is seen from the analysis results that the structural as well as thermal performance of cast carbon steel with an even smaller number of vanes is better than GCI.

Mithlesh et al. (2021) carried out analysis on existing brake discs and modified discs in order to determine maximum deformation, maximum stress, and life span. Disc plate thickness, rotor thickness, and hole diameters were varied on the three discs made of stainless steel. Analysis is carried out to improve stress characteristics and life span using simulation-based ANSYS. The disc with large-diameter holes performed better with reduced stresses and a longer life span due to maximum heat dissipation.

Tauviqirrahman et al. (2023) proposed modifications in the geometry of the brake disc to increase the surface area for maximum heat dissipation. Holes and grooves are made on the surface of the disc. Hole and groove angles were varied, and their effect on heat dissipation was studied. Also, the thermal performance using FEA of various disc materials was assessed. From the results obtained for various materials, it is clear that the grooved disc with a zero-degree hole angle and gray cast iron have the lowest maximum temperature.

5.1 Comparison of different ventilation patterns and materials

Ventilated disc brakes are popular as it offers high heat transfer rates due to an increase in turbulence and greater resistance to thermal deformation due to the uniform distribution of the material. The performance of ventilated disc depends mainly on the geometry of the disc and the optimal configuration of the ventilation channels (García-León et al., 2017). Many researchers have carried thermo-mechanical analysis on ventilated discs considering different design parameters including changes in geometry of disc to expose more surfaces to atmospheric air. The performance of disc can be optimised by providing holes, slits/slots and combination of holes and slits/slots on the surface of the disc. The curved shape of the vanes has great impact as compared to radial vanes on the stresses induced and temperature of the disc (Park et al., 2022; Duzgun, 2012; Afzal, 2020; Dubale et al., 2021; Dhir, 2018; Pranta et al., 2022; Tauviqirrahman et al., 2023). A new concept of a ventilated disc with heat pipes inserted between the two surfaces to carry the heat generated from the frictional surfaces was proposed by Jian et al. (2020). A disc made from conventional material is enclosed with a number of heat pipes made from copper.

Similarly material of the disc is also important from thermal, weight and strength point of view. Researchers have tried different materials which includes martensitic stainless steel and grey cast iron (Seelam et al., 2021), aluminium 7075 T6, titanium grade 5 alloy, and stainless steel 420 (Chheda et al., 2020), carbon–carbon, aluminium, stainless steel and for the titanium alloy (Ahmed et al., 2021), aluminium alloy and nanocomposite aluminium matrix (Sivaprakasam et al., 2022), AISI4140, SS420, and AISI1020 (Choudhary et al., 2023), grey cast iron and cast carbon steel (Al Riyami et al., 2023). A three-dimensional structured silicon carbide foamed material filled with Aluminium metal alloy is also used as a disc material which has the characteristics of withstanding high temperatures with a low wear rate. It can transmit the heat generated at a much faster rate (Nong et al., 2017).

The most popular material widely used is grey cast iron due to its excellent thermophysical and thermoelastic characteristics. Materials with high thermal conductivity, high compressive strength, high wear resistance, high coefficient of friction and high specific heat are the best suitable materials. Cost and weight are also the important parameters for selection the appropriate material.

6 Optimisation of disc design

Performance of disc brake can be improved by the optimisation of different geometrical and operational parameters. Different multi-objective optimisation methods are found in the literature. Taguchi DoE is one of the popular method Besseris (2010). Design optimisation of a ventilated disc was performed by Jafari and Akyüz (2022) considering nine parameters that have an impact on the performance of the disc. The parameters considered were bell link, disc material, ventilation gap, number of fins, fin inlet-outlet radius, fin angle, twist point of fin, and dust shield. The Taguchi method was used to determine the number of experiments. Thermal analysis was then performed on the discs modelled using the optimised parameters to determine the impact of the parameters on the cooling of the disc. While performing the analysis, other components like pads, tyres, rims, and dust shields were also considered. It was concluded that the gap between two friction surfaces has more impact than other parameters on cooling performance.

Grivc et al. (2019) presented the optimisation of the brake disc of rail freight numerically and validated the obtained results by conducting experiments. The material of the disc considered is cast iron. The objective function is to improve heat dissipation and reduce losses in ventilation. After performing step-by-step optimisation, it was found that the ventilation losses were reduced considerably, while there was also a reduction in mass, which can save energy.

Kharate et al. (2018) optimised the disc performance by considering certain operating parameters of the brake disc. Three operating parameters were considered: the magnitude of the pressure, the speed at which the disc rotates, and the normal load acting. A standard design of experiments (DOE) by Taguchi was followed. Four levels for each parameter were decided for Taguchi DOE. The response function was the braking time. After running the simulations, it was discovered that the speed at which the disc rotates has the greatest influence on the braking time, followed by the brake pressure.

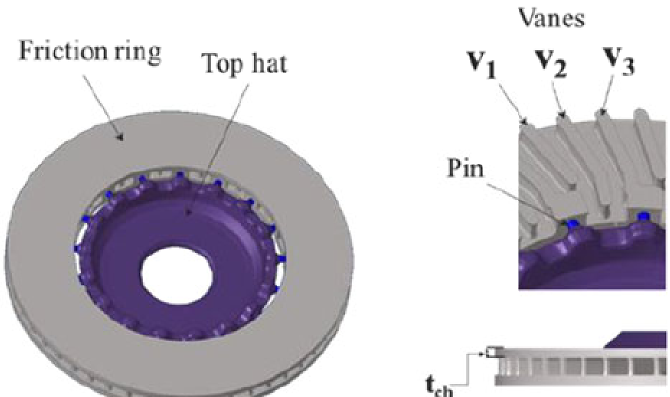
Table 2 Summary of parameters considered and impact parameters in optimisation of disc

Author	*Parameters considered*	*Optimisation method*	*Objective function*	*Impact parameter/s*
Jafari et al. (2022)	Bell link, disc material, ventilation gap, number of fins, fin inlet-outlet radius, fin angle, twist point of fin, and dust shield	Taguchi method	To reduce cooling time	Ventilation gap
Le Gigan (2017)	The cooling vanes and pillar arrangements	Response surface methodology	To increase the fatigue life or to decrease the mass	Cheek thickness and the pillar length
Kharate et al. (2018)	The magnitude of the pressure, the speed at which the disc rotates, and the normal load acting	Taguchi method	To reduce braking time	Speed at which the disc rotates
Barton et al. (2020)	Three different vane thickness and cheek thickness (Figure 9)	Response surface methodology	Minimising mass and improving thermal performance	All
Song and Lee (2009)	The design variables are set up as the thicknesses of three parts in the disc	Kriging metamodels	Minimise stress generated in the disk	All
Pan et al. (2021)	Brake disc single side disc height, heat sink rib height, modulus of elasticity, disc cap height	Response surface methodology	Minimise the weighted brake squeal tendency coefficient	All
Jung et al. (2012)	Five geometrical parameters	Response surface methodology	Minimises both the temperature rise and thermal deformation	All
Belhocine et al. (2021), Kalita et al. (2021)	Inboard plate thickness, outboard plate thickness, vane height, effective offset, and centre hole radius	Taguchi method	Improving fatigue life	All
Zhang et al. (2022)	The total thickness, the height of the ventilation slot, the angle between the sides of the ribs, the number of ribs	Response surface methodology	Minimise the weight and maximum temperature incurred	All
Li et al. (2023)	Number of vanes, vane thickness, and steering angle	Taguchi method	The volume flow of air and heat exchange area	All

Le Gigan (2017) focused on the modifications to the brake disc, emphasising the cooling vanes and pillar arrangements. The cheek thickness and pillar length were important considerations for constructing the cooling arrangement of a brake disc with regard to fatigue, as shown by the RSM utilised for optimisation. On the other hand, the width of the vanes and pillars proved to be of less importance. Optimisations using the response surfaces show a reduction in the mass of the brake disc and a significant improvement in fatigue life compared to the reference brake disc.

Barton et al. (2020) optimised the brake disc using RSM. Coupled thermal-structural analysis using FEA was performed to predict the behaviour of an existing disc for a given braking condition. After performing the analysis, variables affecting the thermo-mechanical performance were identified.

Figure 9 Identified parameters for optimisation (see online version for colours)



Source: Barton et al. (2020)

Four parameters, as shown in Figure 9, were identified: minimising mass and improving thermal performance are taken as the response functions. Optimisation was carried out, and improved dimensions were found that would minimise the mass and maximum temperature.

Song and Lee (2009) introduced a new method of optimisation for reducing the weight of the brake disc known as the kriging interpolation method. The optimisation problem constructed through the kriging interpolation method is solved by an algorithm known as simulated annealing. A new disc is designed for a passenger car, considering friction on the circumferential surface. The results obtained from this new optimisation method are compared with those of ANSYS. Both results are well within their limits.

Pan et al. (2021) optimised the disc brake, considering the problem of brake squeal. The model was validated using a bench test, and based on that, four structural parameters of the disc were selected as dependent variables. The weighted braking squeal tendency coefficient is to be minimised to reduce the problem of brake squeal, and so is the target function. Using RSM, the structural parameters are optimised, analysis is carried out on the disc with optimised parameters, and the results show that there was a reduction of 52.6% in the target function after optimisation. This indicates that by reducing the weighted brake squeal tendency coefficient, the brake squeal can be suppressed significantly.

Generally, the rise in temperature in the disc and deformation are calculated in a very conventional way by first calculating kinetic energy and differentiating it with respect to time to obtain the heat flux, which can be used for further analysis. Jung et al. (2012) developed an analysis technique to overcome the limitations of estimating the rise in temperature and deformation of the disc based on real-time information about the vehicle. The power generated during braking is calculated mathematically, and the heat flux is determined. For the existing disc, the thermal characteristics are determined by performing an analysis. RSM is then used for shape optimisation. Five geometrical parameters with three levels were used as input, while rise in temperature and deformation due to heating are the response functions. As per the results obtained, it can be seen that there is a reduction in the maximum temperature and thermal deformation after optimisation at the same conditions.

Belhocine et al. (2021) presented an optimisation of the brake disc using a novel approach for optimising the design parameters of the ventilated brake disc. Five geometrical parameters identified for optimisation are the thicknesses of the inner and outer discs, the ventilation gap between the inner and outer discs, i.e., vane height, the distance between the inner plate surface and the outer surface, i.e., offset, and the radius of the central hole. These parameters were set as controlling, while fatigue life was the response function. The Taguchi L27 approach to DoE was used, and simulation was carried out. Thermal and structural analysis for determining the fatigue life was performed on the 27 brake disc designs with design parameters obtained through Taguchi DoE. Finally, a genetic algorithm (GA) and particle swarm optimisation (PSO) approach were used to find the values of optimised design parameters. The brake disc of design parameters obtained from algorithms gives improved fatigue life, which is 12.74% higher. The same type of study was carried out by Kalita et al. (2021) to study fatigue failure and predict fatigue life along with the axial deflection of brake discs. The same design parameters were used, but with a different approach. The data obtained from the CCD approach of RSM is used, and a newly developed metaheuristic algorithm known as the grey wolf optimiser is utilised to optimise the parameters of ventilated disc design. The optimisation results provide a drastic improvement over the baseline model and multi-criteria decision model-based solutions.

Zhang et al. (2022) analysed the transient heat transfer in the brake disc under the dynamic convection heat transfer coefficient using FEA. Firstly, a three-dimensional model of the existing disc was analysed for thermal performance, and then the response surface method was used for optimisation. The optimisation variables considered were the geometrical parameters, while the optimisation function was to minimise the weight and maximum temperature incurred. The results obtained show weight reduction as well as a reduction in maximum temperature.

Li and Yang (2023) optimised the radial vane type of brake disc considering geometrical parameters such as the total number of vanes and its thickness, along with the steering angle. The optimisation function was to increase the cooling effect. The Taguchi design of the experiments was used with three levels of each parameter. FEA was used to analyse the air flow and temperature distribution around the disc. At the end of the analysis, the results show that increasing the number of vanes and steering angle can improve the overall cooling effect for a given period of time while driving at a particular speed.

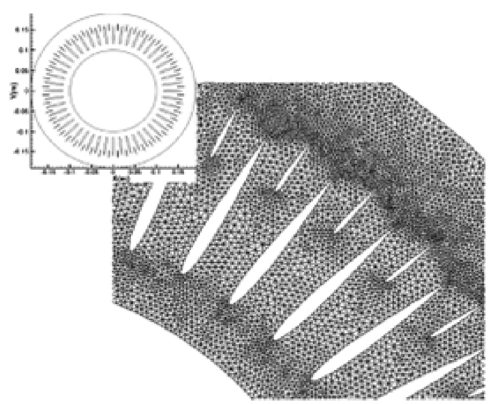
7 Performance improvement using CFD

Lee et al. (2018) carried out a numerical simulation of a ventilated brake disc to examine fluid flow and heat transfer. Along with the brake disc, other geometries associated with the brake assembly, including the wheel, were also considered for analysis. Four cases of a combination of disc and bearing units with different air flows were considered. Using the analysis software, simulations were carried out to study fluid flow, conduction, and convection heat transfer. Investigations of the heat transfer coefficient distribution and flow patterns of cooling air inside the wheel were performed to evaluate the cooling performance in each case. Similar to this, Yan et al. (2017) presented a numerical investigation. For study purposes, a comparison of a standard brake disc and a pin-finned disc with cross-drilled holes is done. A systematic comparison of the heat transfer characteristics and fluid flow was done within a certain range of operating speeds. It was found that, due to the presence of cross-drilled holes on the rubbing surfaces, the performance of the pin-finned disc in terms of cooling capacity is improved. This is due to an increase in pumping capacity.

An increase in the overall heat transfer coefficient increases the cooling effect. Nejat et al. (2011) describe a unique brake disc design that includes airfoil vanes to boost the heat transfer coefficient. The introduction of airfoil vanes increases the mass and flow of air in the ventilation gap. A secondary aerofoil vane in addition to the primary one, as shown in Figure 10, was introduced to improve the ventilating capacity. Finite element analysis is performed, and as per the results, a noted improvement in heat transfer coefficient is found for the new design at different speeds.

Kubota et al. (2000) studied the airflow through the radial vane type of disc along with thermal and structural analysis during braking. The relationship between the parameters, like cooling performance, heat resistance, noise, and vibration, and the weight was studied by performing CFD analysis and actual brake tests. An optimum design of brake disc was proposed that reduces the weight of the disc from 9.22 kg to 7.06 kg with a reduction in squeal index, deformation angle, and disc thickness variation. There was a slight increase in temperature and crack condition, but within the limits.

Figure 10 New designs by adding a secondary airfoil vane



Source: Nejat et al. (2011)

8 Challenges and future directions

Many researchers attempted performance prediction of disc brakes numerically through transient thermal, thermo-mechanical, and structural analysis using commercially available software. Furthermore, few researchers validated the experimental and numerical results. While performing the simulation only convection mode of heat transfer is considered while it is assumed that radiation is insignificant. All modes of heat transfer (conduction, convection and radiation) and accountability of heat absorbed by all the components of the disc brake system should be considered.

Although many studies have been published on ventilation patterns, materials, analysis, and optimisation, researchers have a lot of scope for improvements in the areas of structural performance, wear and fatigue life, squeal and vibrations, etc. Invention of lightweight materials having good thermo-physical properties can lead to better results. Uncertainty analysis is reported by Lü et al. (2018). A case study featuring the random failure probability of a brake disc caused by hot judder was presented by Sim et al. (2022). More studies discussing uncertainty analysis and failure probability are necessary, hence it should be considered. By addressing these future research areas, the research community and designers would advance further in the field of designing an efficient braking system.

9 Conclusions

An attempt has been made to present a systematic review of the literature available on the analysis of disc brakes. Various aspects like ventilation patterns of discs, materials for discs, optimal performance by varying dimensions, air flow simulation, and thermo-mechanical analysis are considered while presenting this review. According to the reported literature, there are a variety of ventilation patterns that can be used to improve air flow and heat dissipation, including straight and curved radial vanes, single and double airfoil vanes, heat pipes inserted between the discs, straight and cross holes drilled on friction surfaces, slots cut to increase surface area, and combinations of holes and slots. Researchers attempted the use of different materials for disc manufacturing, like cast iron, aluminium 7075 T6, titanium grade 5 alloy, stainless steel 420, SiC3D/Al alloy, composite materials such as carbon composites, and aluminium matrix nanocomposite. Despite the numerous efforts made by various researchers, a reasonably priced replacement for cast iron as a disc material has not been discovered. Various optimisation techniques like response surface methodology, Taguchi approach, GA, PSO, and grey wolf optimisation have been reported in the literature for optimisation of disc shape using geometrical parameters like number of vanes, vane thickness, steering angle, vane inlet radius, vane outlet radius, ventilation gap, number of holes, number of slots, inboard and outboard disc thickness, etc.

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